Problem 1

For the following series, write formulas for the sequences a_n, S_n, and R_n, and find the limits of the sequences as $n \to \infty$ (if the limits exist).

$$\sum_1^\infty \frac{1}{2^n}$$

Solution

$$a_n = \frac{1}{2^n}$$

$$S_n = \sum_{i=1}^n \frac{1}{2^i} = \sum_{i=1}^n \left(\frac{1}{2}\right)^i = -1 + \sum_{i=0}^n \left(\frac{1}{2}\right)^i = -1 + \frac{1 - \left(\frac{1}{2}\right)^{n+1}}{1 - \left(\frac{1}{2}\right)} = 1 - 2^{-n}$$

$$S = \lim_{n\to\infty} S_n = \lim_{n\to\infty} (1 - 2^{-n}) = 1$$

$$R_n = S - S_n = 1 - (1 - 2^{-n}) = 2^{-n}$$

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{1}{2^n} = 0$$

$$\lim_{n\to\infty} R_n = \lim_{n\to\infty} 2^{-n} = 0$$